Exam - Calculus B, March 26 2019, 8:30 - 11:30

Some remarks:

-You are not allowed to use books, notes, notebooks, mobile phones, tablets, etc.

-When you are leaving temporarily the room, please hand in your mobile phone to the supervisors.

-You may write in English or in Dutch.

-Don't forget to write your name and your student number on each sheet of paper you are handing in.

-For each answer, explain how you obtained it.

-You may use the formulas: $\sin(2t) = 2\sin(t)\cos(t)$, $\cos(2t) = \cos^2(t) - \sin^2(t)$, $1 = \cos^2(t) + \sin^2(t)$.

Problem 1. *20pts.* Let $f(x, y, z) = e^{x-y^2}\cos(z)$. Calculate

a) ∇f.
b) The directional derivative of f in the direction $(1, 2, 3)$ in the point $(0, 0, 0)$.
c) $\frac{\partial^3 f}{\partial x \partial y \partial z}(x, y, z)$.
d) The second degree Taylor polynomial of f around the point $(0, 0, 0)$.

Problem 2. *15pts.* Let D be the domain in $\mathbb{R}^2$ described by the inequality

$$\frac{x^2}{4} + y^2 \leq 1.$$

a) Draw a sketch of D.
b) Find the minimum and the maximum of the function $h(x, y) = x + 2y$ on D.

Problem 3. *15pts.* Consider the function $g(x, y) = e^{x+y^2}$ and the vector field

$$\mathbf{V} = \frac{-2y}{1+x^2}\mathbf{i} + \frac{1}{1+x^2}\mathbf{j}.$$

a) Sketch the level sets of g.
b) Determine the equation of the line tangent to the level set of g through the point $(3, -1)$.
c) Show that the field lines of the vector field $\mathbf{V}$ coincide with the level sets of the function g.

Problem 4. *20pts.* Let G be the domain in $\mathbb{R}^3$ described by the inequalities

$$0 \leq z \leq x^2 + y^2 + z^2 \leq 4.$$

Calculate

$$\iiint_G \frac{\mathrm{d}x\mathrm{d}y\mathrm{d}z}{x^2 + y^2 + z^2}.$$

Hint: Use spherical coordinates.

Problem 5. *30pts.* Let $\mathcal{S}$ be the surface in $\mathbb{R}^3$ given by

$$x^2 + y^2 = 1, \quad 0 \leq x, \quad 0 \leq y, \quad 0 \leq z \leq y^2.$$

We orient $\mathcal{S}$ such that the positive side of $\mathcal{S}$ is visible from the point $(0, 0, 0)$. On the boundary curve $\mathcal{C}$ of $\mathcal{S}$ we consider the induced orientation. Consider the vector field

$$\mathbf{F} = \cos\left(\frac{x}{x+y}\right)\mathbf{i} + \sin\left(\frac{x}{x+y}\right)\mathbf{j} + e^z(x^2 - y^2)\mathbf{k}.$$

a) Draw a sketch of $\mathcal{S}$. Indicate on the drawing the orientation of $\mathcal{C}$.
b) Calculate $\mathbf{curl}(\mathbf{F})$.
c) Find an oriented parameterization $\mathbf{r} : R \to \mathcal{S}$, and calculate $\mathrm{d}\mathbf{S}$ in this parameterization.
d) Calculate the circulation of $\mathbf{F}$ along $\mathcal{C}$

$$\oint_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}.$$

Hint: use an "integral theorem".